Origin of Scalar Couplings

BCMB/CHEM 8190
Traditional View of Scalar Coupling

Dipole Model: Two equal intensity lines from each interaction for each spin

Reasons to understand couplings:
- magnetization transfer (COSY, HSQC)
- structural information

Measurement: directly as above, 2D J-resolved, intensity based

Problem: dipolar interactions average to zero in solution
Multiple sets of spins

Multiplet structure: as many as $n^2(2^{n-1})$ lines in non-equivalent first order sets, $3 \times 2^2 = 12$ lines for previous example

Unequal intensities in equivalent cases; proton next to a methylene, has states $\alpha\alpha$, $\alpha\beta$, $\beta\alpha$, $\beta\beta$; a 1:2:1 triplet

\[\alpha\alpha,\alpha\beta,\beta\alpha,\beta\beta\]
The Origin of Scalar Coupling Constants

\[ E_J = \sum_{i \neq j} J_{ij} I_i \cdot I_j \approx \sum_{i \neq j} J_{ij} I_{zi} I_{zj} \]

Mechanism:

not through space, but Fermi contact

\[ E \propto (3\cos^2\theta - 1) / r^3 \]

\[ E \propto s_A \times s_B \]

\[ \mu_B \downarrow e^{-} \downarrow \uparrow e^{-} \uparrow \mu_A \]

Low E

\[ \mu_B \uparrow e^{-} \uparrow \downarrow e^{-} \downarrow \mu_A \]

High E

J is positive
Fermi-Contact Expression for Scalar Coupling

• Expression comes from second order perturbation treatment of spin contribution to electronic energy

\[ E = \left(4g\beta\gamma_N h/3\right)^2 \sum_{jk} <\psi_0|\delta(r_{kA})\delta(r_{jX}) S_jz S_kz|\psi_0> I_{zA}I_{zX} \]

• \( I_{zA}I_{zX} \) dependence as expected for \( H_J \)

• \( \gamma_A \gamma_X \) dependence as expected for \( H_J \)

• Depends on electron density at pair of nuclei \( (\delta(r_{kA})\delta(r_{jX}) S_jz S_kz) \) and spins must be correlated – hence bonded

\[ (\int S_{jz}S_{kz} = 0 \text{ if spins are uncorrelated}) \]

• Depends on “s” character of bond

(\( \delta(r_{kA}) \) is only finite for an s orbital)
Predictions about Scalar Couplings

- Wave functions (electronic) for $^1\text{H}-^1\text{H}$ can be calculated at a high level of theory – even at VB prediction of 200Hz is good.

- Experimental $^1\text{H}-^2\text{H}$ is 44 Hz; correcting for $\gamma$s, $J_{\text{H-H}} = \gamma_{\text{1H}}/\gamma_{\text{2H}} \times 44 = 6.5 \times 44 = 265\text{Hz}$

- Directly bonded pairs should have much larger couplings:

\[
\begin{align*}
\text{1H} & \quad \text{11.6 Hz} \quad \text{1H} \\
\text{R} & \quad \text{2.5 Hz vs 265 Hz}
\end{align*}
\]
Some predictions for one-bond couplings

Couplings are short range and depend on electron densities at the coupled nuclei (ie, the product of “s characters”) as well as $\gamma$s.

<table>
<thead>
<tr>
<th>Bond</th>
<th>Frequency</th>
<th>Orbital Type</th>
<th>Predicted Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-C</td>
<td>35 Hz</td>
<td>$sp^3 - sp^3$</td>
<td>$1/4 \times 1/4 = 1/16$</td>
</tr>
<tr>
<td>CH$_3$-CH$_3$</td>
<td>67 Hz</td>
<td>$sp^2 - sp^2$</td>
<td>$1/3 \times 1/3 = 1/9$</td>
</tr>
<tr>
<td>CH$_2$=CH$_2$</td>
<td>57 Hz</td>
<td>$sp^2 - sp^2$</td>
<td>$1/3 \times 1/3 = 1/9$</td>
</tr>
<tr>
<td>C$_6$H$_6$</td>
<td>171 Hz</td>
<td>sp-sp</td>
<td>$1/2 \times 1/2 = 1/4$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bond</th>
<th>Frequency</th>
<th>Orbital Type</th>
<th>Predicted Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-H</td>
<td>125 Hz</td>
<td>s - sp3</td>
<td>$1 \times 1/4 = 1/4$</td>
</tr>
<tr>
<td>CH$_3$-CH$_3$</td>
<td>156 Hz</td>
<td>s - sp2</td>
<td>$1 \times 1/3 = 1/3$</td>
</tr>
<tr>
<td>CH$_2$=CH$_2$</td>
<td>158 Hz</td>
<td>s - sp2</td>
<td>$1 \times 1/3 = 1/3$</td>
</tr>
<tr>
<td>C$_6$H$_6$</td>
<td>249 Hz</td>
<td>s-sp</td>
<td>$1 \times 1/2 = 1/2$</td>
</tr>
</tbody>
</table>
A two-bond coupling is small and sometimes negative – why?

Shared on central carbon

Parallel H and e contributes low energy

Anti parallel H and e contributes high energy

The two terms cancel

Coupling goes up again for a three-bond coupling
Vicinal Couplings ($^3J_{HH}$)

- Originally for H-C-C-H
- M. Karplus (1959) JCP 30 11 - Karplus Equation
  - $J_{HNHA} = 7.0 \cos^2(\phi) - 1.4 \cos(\phi) + 1.7$ for peptides
- (Bax and Wang (1996) JACS 118 2492)
Wang-Bax Equation Compared to Ubiquitin Data

$3J_{HH} (\text{Hz})$

$\phi - 60$

$\phi$ measured as C'-N-C$_{\alpha}$-C'
Populations of Rotomeric States from Couplings
An Application of the Karplus Equation

- Occurs in pyranosides, furanosides, and nucleosides
- Assume only minimum energy states populated
- Assume rapid equilibration – averaging of coupling
- \( J_{56} = 1.8 F_1 + 1.8 F_2 + 9.2 F_3 \)
- \( J_{56}' = 1.8 F_1 + 9.2 F_2 + 1.8 F_3 \)
- \( F_1 + F_2 + F_3 = 1 \)
Application of Karplus Equations (cont)

Measure $J_{65}$ and $J_{6'}5$ for $\beta$-Me-galactose

7.0 Hz, 3.0 Hz

Using averaging formula find:

0.15, 0.7, 0.15 for states 1, 2, and 3

Reference on scalar coupling in Levitt: 217-223
Scalar Couplings Can be Predicted Theoretically


Couplings And Hydrogen Bonds


$^{15}N$-H ......$^{15}N$ or $^{15}N$-H ......O=$^{13}C$

Chemical shifts and Coupling Constants: How Many Peaks can you Assign?