"Biomolecular Nuclear Magnetic Resonance" is a course intended for all graduate students with an interest in applications of nuclear magnetic resonance (NMR) to problems in structural and functional biology. It will begin with a treatment of the fundamentals that underlie magnetic resonance phenomena and develop this into a basis for experimental design, interpretation of data, and critical reading of the literature.

http://tesla.ccrc.uga.edu/courses/bionmr/
Syllabus

I. Introduction
M 1/9  A. Magnetic properties of nuclei and electrons - precession  5-38 L*
W 1/11  B. RF pulses and spin relaxation - Bloch equations  39-50 L*, 653 L*

II. Instrumentation
M 1/16  MLK Jr. Holiday (no class)
W 1/18  A. Instrumental considerations - a look at probes  65-76L*
M 1/23  B. Fourier transform methods and data Processing  85-102 L*, 78-101 K*

- Friday Labs are scheduled separately (C122 Davison Life Sciences) – intro to computer systems 1/13, 1/20
- Texts:
  - “Spin Dynamics - Basics of Nuclear Magnetic Resonance” (2nd edition), M. H. Levitt (L)*
  - “Understanding NMR Spectroscopy” (2nd edition), J. Keeler (K)*
Biomolecular NMR 2014

- Biomolecular NMR - short history ~ 1985 first protein structure
- Compared to X-ray ~ 1953 first protein structure
- Today ~ 11 % of structures in the PDB (10,287) come via NMR – higher for nucleic acids
- Unique structural applications – weak associations, partially structured, membrane associations, in-cell observation
- Diverse applications: drug screening, metabolic monitoring, in vivo imaging
- NMR is still an evolving science
NMR Recognition

• 1944 – Isidor Isaac Rabi - Nobel Prize in Physics
  -"for his resonance method for recording the magnetic properties of atomic nuclei"

• 1952 – Felix Bloch and Edward Mills Purcell – Nobel Prize in Physics
  -"for their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith"

• 1991 – Richard Ernst – Nobel Prize in Chemistry
  -"for his contributions to the development of the methodology of high resolution nuclear magnetic resonance (NMR) spectroscopy"

• 2002 – Kurt Wuthrich – Nobel Prize in Chemistry
  -"for his development of nuclear magnetic resonance spectroscopy for determining the three-dimensional structure of biological macromolecules in solution"

• 2003 – Paul Lauterbur and Sir Peter Mansfield – Nobel Prize in Physiology and Medicine
  -"for their discoveries concerning magnetic resonance imaging"
Varian HR 220

~1965
Superconducting Magnets Boosted Field Strength Required a Lot of Care And Feeding
High Field (220 MHz), but Still 1D CW NMR
~1970 Richard Ernst Introduced Multidimensional Pulse FT Methods
Fourier transform (top) and conventional spectra of 0.011 M progesterone showing sensitivity enhancement by a factor ten.
~1982 Kurt Wüthrich: 2D $^1\text{H}-^1\text{H}$ NMR: ~10 kDa Protein assign resonances, collect NOE’s, calculate structure
Extension to 3D: Through-bond Correlations in Peptides
Isotope Labeling is Key

Today Very Large Systems Can Be Studied: Proteasome subunit – active site dynamics


Ruschak AM and Kay LE, 2010, Methyl groups as probes of supra-molecular structure, dynamics and function, J Biomol NMR **46**:75-87
Example: Fumaric acid to malic acid conversion indicates onset of acute tubular necrosis of the mouse kidney. Images are 10 and 18 hrs after folic acid induced nephropathy. Left and right images based on signals of carboxyl resonances of fumaric and malic acid respectively.
NMR is widely applicable to structure and function of biomolecules


NMR Active Isotopes Exist for Nearly Every Element

http://bouman.chem.georgetown.edu/NMRpt/NMRPerTab.html

Select an element by clicking on it:

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>Li</td>
<td>Be</td>
<td>B</td>
<td>C</td>
<td>N</td>
<td>O</td>
<td>F</td>
<td>Ne</td>
<td>Al</td>
<td>Si</td>
<td>P</td>
<td>S</td>
<td>Cl</td>
<td>Ar</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Na</td>
<td>Mg</td>
<td>Mg</td>
<td>K</td>
<td>Ca</td>
<td>Sc</td>
<td>Ti</td>
<td>V</td>
<td>Cr</td>
<td>Mn</td>
<td>Fe</td>
<td>Co</td>
<td>Ni</td>
<td>Cu</td>
<td>Zn</td>
<td>Ga</td>
<td>Ge</td>
<td>As</td>
</tr>
<tr>
<td>Rb</td>
<td>Sr</td>
<td>Y</td>
<td>Zr</td>
<td>Nb</td>
<td>Mo</td>
<td>Tc</td>
<td>Ru</td>
<td>Rh</td>
<td>Pd</td>
<td>Ag</td>
<td>Cd</td>
<td>In</td>
<td>Sn</td>
<td>Sb</td>
<td>Te</td>
<td>I</td>
<td>Xe</td>
</tr>
<tr>
<td>Cs</td>
<td>Ba</td>
<td>*</td>
<td>Hf</td>
<td>Ta</td>
<td>W</td>
<td>Re</td>
<td>Os</td>
<td>Ir</td>
<td>Pt</td>
<td>Au</td>
<td>Hg</td>
<td>Tl</td>
<td>Pb</td>
<td>Bi</td>
<td>Po</td>
<td>At</td>
<td>Rn</td>
</tr>
<tr>
<td>Fr</td>
<td>Ra</td>
<td>**</td>
<td>Rf</td>
<td>Ha</td>
<td>Sg</td>
<td>Ns</td>
<td>Hs</td>
<td>Mt</td>
<td>**</td>
<td>Ac</td>
<td>Th</td>
<td>Pa</td>
<td>U</td>
<td>Np</td>
<td>Pu</td>
<td>Am</td>
<td>Cm</td>
</tr>
</tbody>
</table>
Review of Spin Properties

- NMR active nuclei possess an intrinsic angular momentum, $\vec{I}$, known as the spin angular momentum. The magnitude is
  $$\vec{I} = \hbar \left[ I \ (I+1) \right]^{1/2}$$

- Here $I$ is the nuclear spin quantum number (integral or half-integral). If $I = 0$, no spin angular momentum (not NMR active)

- Associated with $I$ is a magnetic moment, $\mu$
  $$\vec{\mu} = \gamma \vec{I} = \gamma \hbar \left[ I \ (I+1) \right]^{1/2}$$

- The proportionality constant is the gyromagnetic ratio, $\gamma$

- In NMR, larger $\vec{\mu}$ for given $\vec{I}$ (large $\gamma$), means more sensitive nucleus
In a magnetic field, otherwise degenerate (energetically equivalent) states split into nondegenerate states (known as Zeeman splitting).

The states are quantized, with the number of states established by the spin quantum number, $l$

$$\# \text{ levels} = 2l + 1$$

Each of the $2l+1$ states/levels is associated with a magnetic quantum number, $m$

$$m = -l, -l+1, \ldots, l-1, l$$

The component of $l$ along the $z$ axis, $l_z$, is defined as follows

$$l_z = m\hbar$$

Thus

$$\mu_z = \gamma l_z = m\gamma \hbar$$
Review of Spin Properties

- The energies of the states resulting from the interaction of the magnetic moment with a magnetic field, $\vec{B}$ are given by
  $$E = -\vec{\mu} \cdot \vec{B}$$
- The energies of the states depend on the orientations of the moments in the magnetic field, hence are proportional to the scalar projection of $\vec{\mu}$ on $\vec{B}$ (the dot product), $\mu_z$
  $$E = -\mu_z B_0 = -m \gamma \hbar B_0$$
- Here $B_0$ is the magnetic field strength
- The $2I+1$ energy levels are equally spaced. The energy difference between any two adjacent levels is
  $$\Delta E = \gamma \hbar B_0$$
Review of Spin Properties

• The torque exerted by $B_0$ on the magnetic moments/dipoles promotes precession about the $z$-axis at a frequency given by
  \[ \nu_L = \frac{\gamma B_0}{2\pi} \text{ (Larmor frequency, in Hz)} \]
  \[ \omega_0 = \gamma B_0 \text{ (radians/sec)} \]

• The energy difference between energy (spin) states can then be written as
  \[ \Delta E = h\nu_L \]

• Transitions between energy (spin) states can be effected by an electromagnetic field with an energy equal to $\Delta E$. This occurs when the frequency of that field, $\nu_1$, is equal to the Larmor frequency (resonance condition).
  \[ \nu_1 = \nu_L \]
For spin 1/2 \((I = 1/2)\), there are \(2I+1 = 2\) energy levels, with values of \(m\) equal to +1/2 and -1/2, called \(\alpha\) and \(\beta\), with energies

\[ E_\alpha = -\frac{1}{2} \gamma \hbar B_0 \quad E_\beta = +\frac{1}{2} \gamma \hbar B_0 \]

From Boltzmann statistics, the population ratio of these states can be estimated

\[ \frac{N_\beta}{N_\alpha} = \exp\left(\frac{-\Delta E}{k_B T}\right) \approx 1 - \left(\frac{\Delta E}{k_B T}\right) \approx 1 - \left(\frac{\gamma \hbar B_0}{k_B T}\right) \]

- example: \(^1\text{H}, 300 \, ^\circ\text{K}, 5.875 \, \text{Tesla} \, (250 \, \text{MHz})\)

\[ \frac{N_\beta}{N_\alpha} = 1 - \frac{26.7519 \times 10^7 \times 1.0546 \times 10^{-27} \times 5.875}{1.3805 \times 10^{-16} \times 300} = 0.99996 \]

\(\Delta E\) is small, so the populations of \(\alpha\) and \(\beta\) are nearly equal, and the macroscopic magnetization is small: \(\text{NMR is insensitive}\)
Review of Spin Properties

- The sum of the $z$-components of the nuclear dipoles in an ensemble gives the macroscopic (bulk) magnetization, $M_0$

$$M_0 = \gamma \hbar \sum_{m=-I}^{I} m N_m \quad \text{(recall } \mu_z = m \gamma \hbar)$$

$$M_0 \approx \frac{N \gamma^2 \hbar^2 B_0}{k_B T (2I+1)} \sum_{m=-I}^{I} m^2 \approx \frac{N \gamma^2 \hbar^2 B_0 I(I+1)}{3k_B T}$$

- Note: dependence on $\gamma^2$, linear dependence on $B_0$, dependence on isotopic abundance ($N$)
Spin ½ Nuclei are Most Useful in Biomolecular NMR

<table>
<thead>
<tr>
<th></th>
<th>$^1$H</th>
<th>$^{13}$C</th>
<th>$^{15}$N</th>
<th>$^{19}$F</th>
<th>$^{31}$P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>Natural abundance</td>
<td>99.985%</td>
<td>1.108%</td>
<td>0.37%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Magnetogyric ratio ($\gamma/10^7$, rad T$^{-1}$ s$^{-1}$)</td>
<td>26.7519</td>
<td>6.7283</td>
<td>-2.7126</td>
<td>25.1815</td>
<td>10.8394</td>
</tr>
<tr>
<td>Relative sensitivity</td>
<td>1.00</td>
<td>$1.59 \times 10^{-2}$</td>
<td>$1.04 \times 10^{-3}$</td>
<td>0.83</td>
<td>$6.63 \times 10^{-2}$</td>
</tr>
<tr>
<td>Relative receptivity</td>
<td>1.00</td>
<td>$1.76 \times 10^{-4}$</td>
<td>$3.85 \times 10^{-6}$</td>
<td>0.83</td>
<td>$6.63 \times 10^{-2}$</td>
</tr>
<tr>
<td>Magnetic moment ($\mu/\mu_N$)</td>
<td>4.8372</td>
<td>1.2166</td>
<td>-0.4903</td>
<td>4.5532</td>
<td>1.9601</td>
</tr>
<tr>
<td>Quadrupole moment</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Resonance frequency (MHz)</td>
<td>100</td>
<td>25.144</td>
<td>10.133</td>
<td>94.077</td>
<td>40.481</td>
</tr>
</tbody>
</table>
Polypeptides are Rich in NMR Active Nuclei

- Amide $^{1}H$
- Amide $^{15}N$
- Carboxyl $^{13}C$
- Cα $^{1}H$
- Cβ $^{13}C$
- Methyl $^{1}H$
- Cα $^{13}C$
Nuclear Properties

• Not all nuclei have magnetic moments, Why?
• Not all nuclei are equally abundant, Why?
• Spins vary, Why?
• Magnetogyric ratios vary, Why?
Fundamental Particle Properties

Stern Gerlach experiment:

- demonstrated particles (electrons) possess an intrinsic angular momentum, and it is quantized

- Na atom - 1 unpaired electron
  Two spots implies quantized moments: +/- 1/2
  protons and neutrons are also spin 1/2 particles
Understanding Magnetic Moments

- Current Loop Model: classical analogy to connect “spin” to magnetic moment
- Can get reasonable estimate of $\gamma$ for electron

Estimates: $i = -ev/(2\pi r)$, $S = \pi r^2$, $M$ (or $\mu$) = $-erv/2$

$\mu = -e(r \times v)/2$, $\overrightarrow{L} = m_e \overrightarrow{r} \times \overrightarrow{v}$, $\mu = -e/(2m_e) \overrightarrow{L} = \gamma \overrightarrow{L} = \gamma h/(2\pi)l$

$\gamma = -g \left(e/(2m_e)\right)$, $g = $ Lande g factor
Values of Particle Magnetogyric Ratios

Electron: \( g \approx 2, \quad \gamma_e = -17.7 \times 10^{10} \text{T}^{-1}\text{s}^{-1} \)

Proton: expect \( 1/m_p \) dependence, \( 1/2000 \) and positive \( 2.7 \times 10^8 \text{T}^{-1}\text{s}^{-1} \)

Neutron: similar mass to proton \( -1.8 \times 10^8 \text{T}^{-1}\text{s}^{-1} \)
Heavier Nuclei: the Shell Model

Analogous to shell model for atomic electrons

Some rules:

a) spherical particle in a box potential
\[ \psi = R_{nl}(r) \, Y^m_l(\theta,\phi), \quad E(n,l) \]
ladder of energy levels like H atom, but all l's allowed
\[ l=0, 1, 2, 3 \] for “s”, “p”, “d”, and “f” like atomic case

b) strong coupling of spin and orbit angular momentum
quantized total: \[ j = l \pm 1/2 \] for spin 1/2 particle
larger j, lower energy (usually)
c) Treat protons and neutrons separately and fill from bottom up assuming $2j + 1$ degeneracy

d) Assume particle pair strongly within levels: only unpaired spins count - total spin angular momentum given by $j$ of level for unpaired spin

e) Sign of moment depends on sign of moment for fundamental particle ($\pm \frac{1}{2}$ for proton, $-\frac{1}{2}$ for neutron) but changes sign when moment subtracts instead of adds to $l$ in giving $j$
Energy Level Diagram

\[
\begin{align*}
n + 1 & \quad j & \quad \text{degeneracy} & \quad \text{total} \\ & (j = l \pm \frac{1}{2}) & (2j + 1) \\
2s \ (l=0) & 1/2 & 2 & 20 \\
1d \ (l=2) & 3/2 & 4 & \\
1p \ (l=1) & 1/2 & 2 & 8 \\
1s \ (l=0) & 1/2 & 2 & 2
\end{align*}
\]
Example: $^{13}_6\text{C}$ (6 protons, 7 neutrons)
- unpaired neutron (-1/2) in $1p_{1/2}$ ($j = 1 - 1/2 = 1/2$), so spin=1/2, positive $\gamma$

<table>
<thead>
<tr>
<th>$n+1$</th>
<th>protons</th>
<th>neutrons</th>
<th>$j$</th>
<th>degeneracy</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2s ($l=0$)</td>
<td>_</td>
<td>_</td>
<td>1/2</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>1d ($l=2$)</td>
<td>_ _ _ _ _ _ _</td>
<td>_ _ _ _ _ _ _</td>
<td>3/2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>1p ($l=1$)</td>
<td>_ _ _ _ _ _ _</td>
<td>_ _ _ _ _ _ _</td>
<td>3/2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>1s ($l=0$)</td>
<td>_ _ _ _ _ _ _</td>
<td>_ _ _ _ _ _ _</td>
<td>1/2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
Example: $^{15}_7\text{N}$ (7 protons, 8 neutrons)
- unpaired proton (+1/2) in 1p$_{1/2}$ ($j = 1 - 1/2 = 1/2$), so spin=1/2, negative $\gamma$

<table>
<thead>
<tr>
<th>$n+1$</th>
<th>protons</th>
<th>neutrons</th>
<th>j</th>
<th>degeneracy</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2s ($l=0$)</td>
<td>1/2</td>
<td>2</td>
<td>1/2</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>1d ($l=2$)</td>
<td>3/2</td>
<td>4</td>
<td>5/2</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>1p ($l=1$)</td>
<td>1/2</td>
<td>2</td>
<td>3/2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>1s ($l=0$)</td>
<td>1/2</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example: $^{16}_{8}$O (8 protons, 8 neutrons, two magic numbers), spin = 0
- highly stable (99.76% of all oxygen on Earth)

<table>
<thead>
<tr>
<th>n+1</th>
<th>protons</th>
<th>neutrons</th>
<th>j (j = l ± $\frac{1}{2}$)</th>
<th>degeneracy</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2s (l=0)</td>
<td>2/2</td>
<td>2/2</td>
<td>1/2</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>1d (l=2)</td>
<td>3/2</td>
<td>3/2</td>
<td>3/2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>1p (l=1)</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>1s (l=0)</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
Proton Spin Mystery Gains a New Clue: